Research Inventy: International Journal of Engineering And Science
Vol.4, Issue 8 (August 2014), PP 87-90
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

A generalized bernoulli sub-ODE Method and Its applications for nonlinear evolution equation

[1]Yuejun Pan, [2,*]Qinghua Feng
[1](*School of Science, Shandong University of Technology, Zibo, Shandong, China*)
[2](*School of Science, Shandong University of Technology, Zibo, Shandong, China*)

ABSTRACT : *In this paper, with the aid of the mathematical software Maple, we derive explicit exact travelling wave solutions for the Fitzhugh-Nagumo equation by the generalized Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear evolution equations, and can be applied to solve many other nonlinear evolution equations.*

KEYWORDS: *Bernoulli sub-ODE method, travelling wave solutions, exact solution, evolution equation, Fitzhugh- Nagumo equation*

I. INTRODUCTION

During the past four decades or so searching for explicit solutions of nonlinear evolution equations by using various different methods have been the main goal for many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform, the Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion meth-od and so on [1-13].

In [14], Wang presented a (G'/G) expansion method, which are payed much attention by many authors. The method is a typical case of sub-ODE methods.. In this paper, we apply the Bernoulli sub-ODE method [15-16] to construct exact travelling wave solution-ns for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to find exact travelling wave solutions of the Fitzhugh-Nagumo equation. In the last Section, some conclusions are presented.

II. DESCRIPTION OF THE BERNOULLI SUB-ODE METHOD

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2 \tag{1}$$

where $\lambda \neq 0, G = G(\xi)$.

When $\mu \neq 0$, Eq. (1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + d e^{\lambda \xi}} \tag{2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} \ldots\ldots) = 0 \tag{3}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1.We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{4}$$

The travelling wave variable (4) permits us reducing Eq. (3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', \ldots\ldots) = 0 \tag{5}$$

Step 2. Suppose that the solution of (5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + \ldots\ldots \tag{6}$$

where $G = G(\xi)$ satisfies Eq. (1), and $\alpha_m, \alpha_{m-1}\ldots$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in (5).

Step 3. Substituting (6) into (5) and using (1), collecting all terms with the same order of G together, the left-hand side of Eq. (5) is converted into another polynomial in G . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, \ldots\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (1), we can construct the travelling wave solutions of the nonlinear evolution equation (5).

In the subsequent sections we will illustrate the validity of the proposed method by applying it to solve several nonlinear evolution equations.

In the subsequent sections we will illustrate the proposed method in detail by applying it to Fitzhugh-Nagumo equation

.

III. APPLICATION OF THE BERNOULLI SUB-ODE METHOD FOR FITZHUGH-NAGUMO EQUATION

In this section, we will consider the following Fitzhugh-Nagumo equation:

$$u_t - u_{xx} = u(u-\alpha)(1-u) \tag{7}$$

Suppose that

$$u(x,y,t) = u(\xi), \xi = k(x-ct) \tag{8}$$

where c, k are constants that to be determined later.

By (8), (7) is converted into an ODE

$$-cku - ku'' + u^3 - (1+\alpha)u^2 + u\alpha = 0 \tag{9}$$

Suppose that the solution of (9) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{10}$$

where a_i are constants, and $G = G(\xi)$ satisfies Eq. (1).

Balancing the order of u^3 and u'' in Eq.(9), we have $3m = m + 2 \Rightarrow m = 1$ .So Eq.(10) can be rewritten as

$$u(\xi) = a_1 G + a_0, a_1 \neq 0, \tag{11}$$

where a_1, a_0 are constants to be determined later.

Substituting (11) into (9) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0 : -cka_0 + a_0^3 + \alpha a_0 - (1+\alpha)a_0^2 = 0$$

$$G^1 : \alpha a_1 + 3a_0^2 a_1 - ka_1\lambda^2 - cka_1 - 2(\alpha+1)a_0 a_1 = 0$$

$$G^2 : 3a_0 a_1^2 - \alpha a_1^2 + 3ka_1\mu\lambda - a_1^2 = 0$$

$$G^3 : -2a_1 k\mu^2 + a_1^3 = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_1 = -\frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}, a_0 = \frac{2}{3}(1+\alpha),\ c = -\frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}, k = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2} \tag{12}$$

where a_0 is an arbitrary constants.

Substituting (12) into (11), we get that

$$u_1(\xi) = -\frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}G + \frac{2}{3}(1+\alpha) \tag{13}$$

where $\xi = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x + \frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}t]$.

Combining with Eq. (2), we can obtain the travelling wave solutions of (7) as follows:

$$u_1(\xi) = -\frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + \frac{2}{3}(1+\alpha)$$

where a_0, d are arbitrary constants, and $\xi = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x + \frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}t]$.

Furthermore we have

$$u_1(x,t) = -\frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x+\frac{1}{2}\frac{\lambda^2(-5\alpha+2\alpha^2+2)}{(1+\alpha)^2}t]}}\right) + \frac{2}{3}(1+\alpha) \tag{14}$$

Case 2:

$$a_1 = \frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}, a_0 = 0,\ c = -\frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}, k = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2} \tag{15}$$

where a_0 is an arbitrary constants.

Substituting (15) into (11), we get that

$$u_2(\xi) = \frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}G \tag{16}$$

where $\xi = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x + \frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}t]$.

Combining with Eq. (2), we can obtain the travelling wave solutions of (7) as follows:

$$u_2(\xi) = \frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)$$

where $\xi = \frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x + \frac{1}{2}\frac{\lambda^2(-5\alpha + 2\alpha^2 + 2)}{(1+\alpha)^2}t]$.

Furthermore we have

$$u_{12}(x,t) = \frac{2}{3}\frac{\mu(1+\alpha)}{\lambda}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\frac{2}{9}\frac{(1+\alpha)^2}{\lambda^2}[x+\frac{1}{2}\frac{\lambda^2(-5\alpha+2\alpha^2+2)}{(1+\alpha)^2}t]}}\right) + \frac{2}{3}(1+\alpha) \tag{17}$$

Remark: Our result (14) and (17) are new families of exact travelling wave solutions for Eq. (7).

IV. CONCLUSIONS

We have seen that some new travelling wave solutions of Fitzhugh-Nagumo equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the travelling wave variable as well as integrating can be expressed by an m -th degree polynomial in G , where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also this method can be used to many other nonlinear problems.

REFERENCES

[1] M. L. Wang, Solitary wave solutions for variant Boussinesq equations, *Phys. Lett. A, 199,* 1995, 169-172.

[2] E. M. E. Zayed, H. A. Zedan, K. A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, *Chaos, Solitons and Fractals, 22,* 2004, 285-303.

[3] L. Yang, J. Liu, K. Yang, Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, *Phys. Lett. A, 278,* 2001, 267-270.

[4] E. M. E. Zayed, H. A. Zedan, K. A. Gepreel, Group analysis. and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations, *Int. J. Nonlinear Sci. Numer. Simul., 5,* 2004, 221-234.

[5] M. Inc, D. J. Evans, On travelling wave solutions of some nonlinear evolution equations, *Int. J. Comput. Math., 81,* 2004, 191-202.

[6] M. A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, *Appl. Math. Comput., 190,* 2007, 988-996.

[7] M. J. Ablowitz, P. A. Clarkson, *Solitons, Non-linear Evolution Equations and Inverse Scattering Transform* (Cambridge University Press, Cambridge, 1991).

[8] M. R. Miura, *Backlund Transformation* (Springer-Verlag, Berlin, 1978).

[9] C. Rogers, W.F. Shadwick, *Backlund Transformations* (Academic Press, New York, 1982).

[10] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, *J. Math. Phys., 14,* 1973, 805-810.

[11] R. Hirota, J. Satsuma, Soliton solution of a coupled KdV equation, *Phys. Lett. A, 85,* 1981, 407-408.

[12] A. V. Porubov, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, *Phys. Lett. A, 221,* 1996, 391-394.

[13] Z. Y. Yan, H.Q. Zhang, New explicit solitary wave solutions and periodic wave solutions for Whitham-Broer-Kaup equation in shallow water, *Phys. Lett. A, 285,* 2001, 355-362.

[14] M. L. Wang, X. Z. Li, J. L. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. *Phys. Lett. A, 372,* 2008, 417-423.

[15] C. B. Tan, Q. H. Feng, EXACT TRAVELLING WAVE SOLUTIONS FOR THREE NONLINEAR EVOLUTION EQUATIONS BY A BERNOULLI SUB-ODE METHOD. *International Journal of Research and Reviews in Applied Sciences, 16(2),* 2013, 235-240.

[16] H. L. Fan, B. Zheng, SOME NEW TRAVELLING WAVE SOLUTIONS FOR THE COUPLED KDV EQUATION AND THE (2+1) DIMENSIONAL PKP EQUATION. *International Journal of Research and Reviews in Applied Sciences, 16(3),* 2013, 356-359.